Assignment for Class XII

Mathematics

Topic : Determinant (continued)

General Direction for students: whatever the questions solved, everything must be copied in the Maths copy and then do the Homework in the same copy.

Homogeneous system of Linear Equations

A system of linear equation AX=B is sad to be homogeneous if B=O(null matrix)

The homogeneous system of linear equation AX=O has

- only *trivial solution* if |A|≠0; (The trivial solution is x=y=z=0)
- *Infinitely many solutions*, if |A|=0, (To be verified)

Ex 4.5 Q13.i). 5x +5y +2z=0 ,2x +5y +4z=0, 4x +5y +2z=0

The given system of equation can be written as

$$\begin{bmatrix} 5 & 5 & 2 \\ 2 & 5 & 4 \\ 4 & 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ ie AX=O

Where A=$\begin{bmatrix} 5 & 5 & 2 \\ 2 & 5 & 4 \\ 4 & 5 & 2 \end{bmatrix}$, X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ O=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now ,|A|=$\begin{vmatrix} 5 & 5 & 2 \\ 2 & 5 & 4 \\ 4 & 5 & 2 \end{vmatrix}$ =5(-10)-5(-12) +2(-10) =-50 +60-20=-10 ≠0

$\Rightarrow$ the given system has only the trivial solution ie x=0, y=0 & z=0

Ex 4.5 Q14.i). 2x-3y-z=0 , x+3y -2z =0 , x-3y=0

The given system of equation can be written as

$$\begin{bmatrix} 2 & -3 & -1 \\ 1 & 3 & -2 \\ 1 & -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ ie AX=O

Where A=$\begin{bmatrix} 2 & -3 & -1 \\ 1 & 3 & -2 \\ 1 & -3 & 0 \end{bmatrix}$, X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ O=$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & -3 & -1 \\ 1 & 3 & -2 \\ 1 & -3 & 0 \end{vmatrix}$ = 2(-6) +3(2) -1(-6) = -12 +6+6=0

$\Rightarrow$ the given system has infinitely many solution (to be verified also)

For solution of give system ;

let z=k , where k is any real number & from first two equation of the system, we get

2x -3y=k , x+ 3y =2k & this system can be written as AX=B

Where A=$\begin{bmatrix} 2 & -3 \\ 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and B=$\begin{bmatrix} k \\ 2k \end{bmatrix}$

Now , |A| = 6+3=9 ≠0

$\Rightarrow$ these equations have a unique solution $X=A^{-1}B$

$\therefore$ *adj A* =$\begin{bmatrix} 3 & 3 \\ -1 & 2 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{9}\begin{bmatrix} 3 & 3 \\ -1 & 2 \end{bmatrix}$

$\therefore$ $X = A^{-1}B = \frac{1}{9}\begin{bmatrix} 3 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} k \\ 2k \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9k \\ 3k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ \frac{k}{3} \end{bmatrix}$

$\Rightarrow$ x=k , y = $\frac{k}{3}$, z=k

Note that these values of x,y and z also satisfy third equation

$\Rightarrow$x -3y =k -3($\frac{k}{3}$) = k -k=0

$\therefore$ x=k , y = $\frac{k}{3}$ and z=k , where k is any number

Homework: Exercise 4.5 Q13 ii), 14ii) and From outside the book

1). 2x +3y =0, 5x + y=0 2) , 7x +4y=0 , 21x +12y=0 , 3).x+y-2z=0, 2x +y -3z=0, 5x +4y -9z=0